Synthesizing Abstract Transformers

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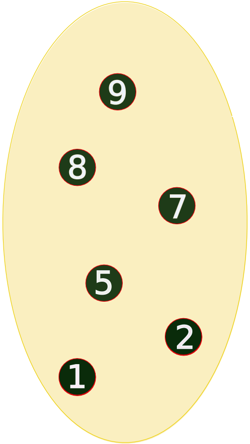
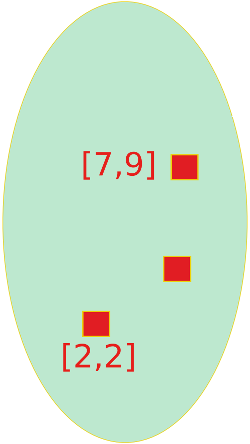
Abstract Interpretation

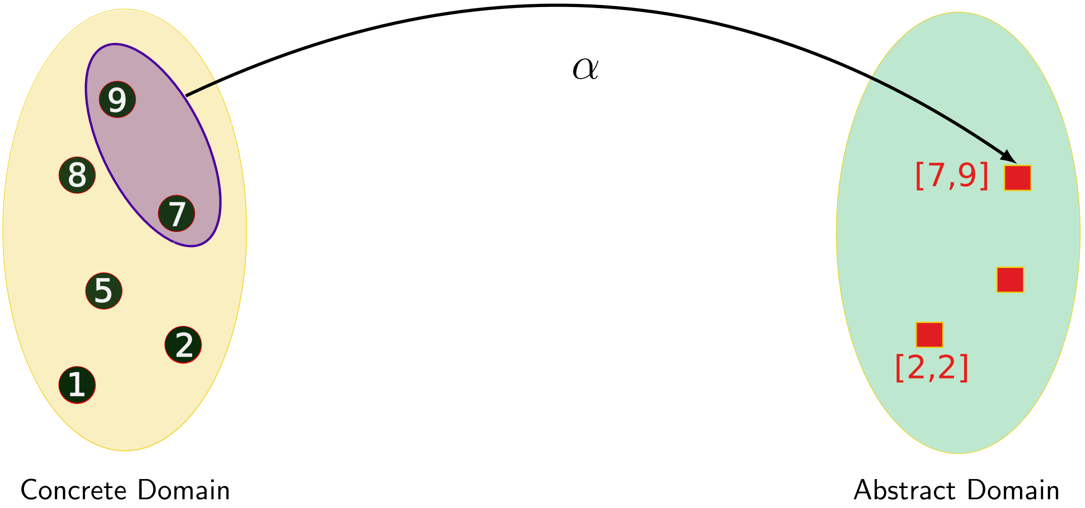


Concrete Domain



Abstract Domain

Abstract Interpretation



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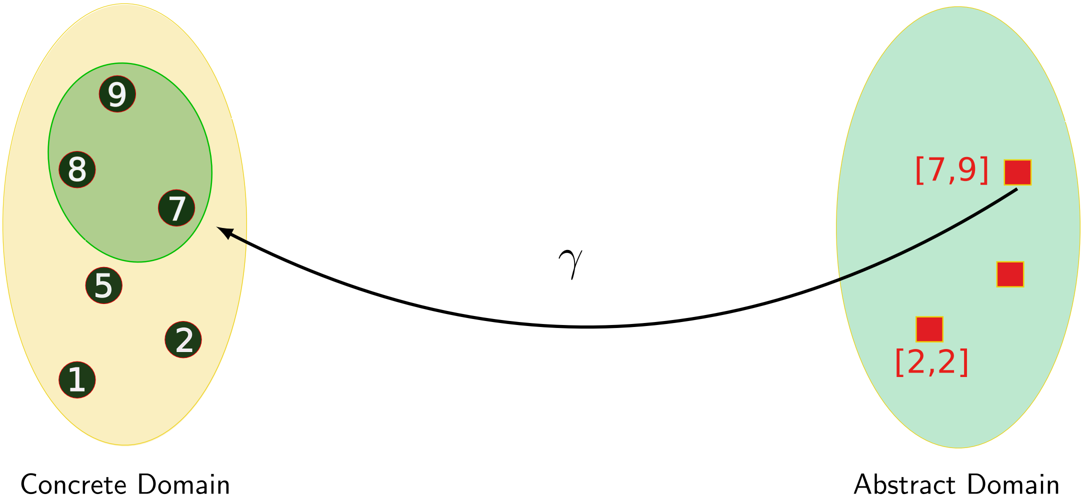

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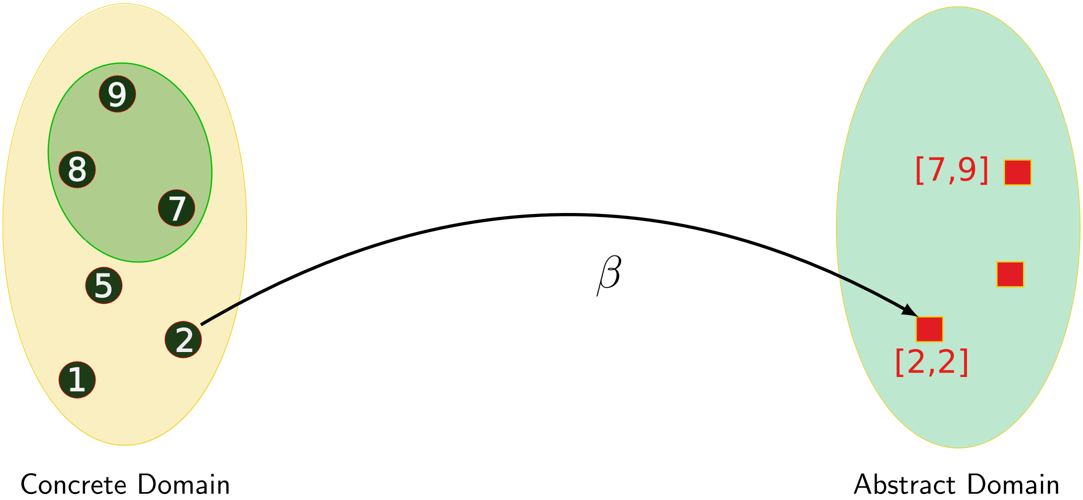

Abstract Transformer

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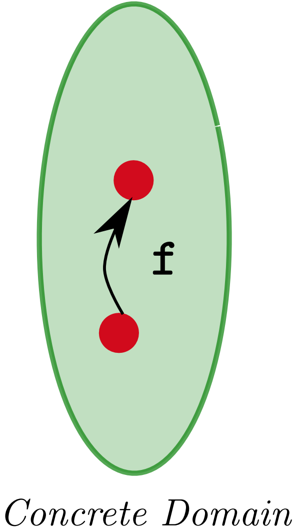
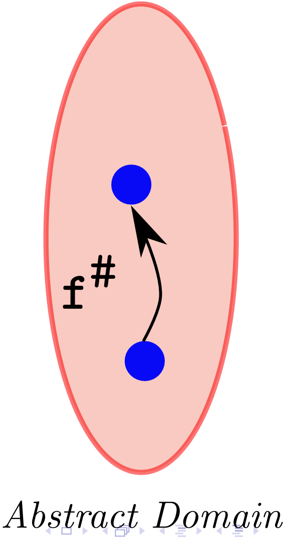


- Tricky even for trivial operation
- Error-prone



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Given the

- concrete semantics Φ_f of a concrete transformer f,
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$$\widehat{f^\sharp} = \lambda a : \sqcup\{\beta(f(c_i)) \mid c_i \in \gamma(a)\}$$

Motivation

Given the following domain-specific language, we desire to get an abstract transformer for Math.abs in the interval domain:

$$\mathit{Transformer} ::= \lambda \texttt{a}.[E, E]$$
$$E ::= \texttt{a.l} \mid \texttt{a.r} \mid 0 \mid -E \mid +\infty \mid -\infty \mid E + E \mid E - E \mid E * E \mid \texttt{min}(E, E) \mid \texttt{max}(E, E)$$

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Given the following domain-specific language, we de bstract transformer for Math.abs in the interval domain:

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We could emit one of the following abstract transformers:

$$\mathtt{abs}^{\sharp}_{1}(\mathtt{a} : \mathcal{A}_{\mathtt{intv}}) : \mathcal{A}_{\mathtt{intv}} = [\mathtt{max}(\mathtt{max}(0, \mathtt{a.l}), -\mathtt{a.r}),\ \mathtt{max}(-\mathtt{a.l}, \mathtt{a.r})]$$

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$$\mathsf{abs}_2^\sharp(\mathsf{a} : \mathcal{A}_{\texttt{intv}}) : \mathcal{A}_{\texttt{intv}} = [\texttt{max}(0, \mathsf{a}.\texttt{l}) - \texttt{min}(0, \mathsf{a}.\texttt{r}),\ \texttt{max}(-\mathsf{a}.\texttt{l}, \mathsf{a}.\texttt{r})]$$

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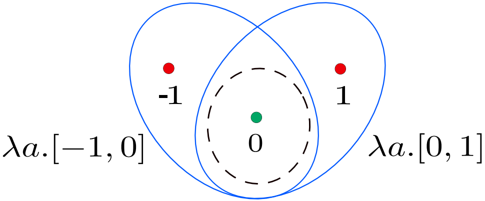
2. $\widehat{f^\sharp}$ may not be expressible in L.
3. Precision defines a partial ordering on abstract transformers, so $f^\sharp \in L$ may not be unique. Example:

`f(x) = 0,`
$L = \{\lambda a.\ [0, k], \lambda a.\ [-k, 0]\ \mid k \in \mathbb{N} \wedge k \geq 1\}$,
$\widehat{\mathcal{S}}^\sharp_L = \{\lambda a.[0,1], \lambda a.[-1,0]\}$



Best L-transformer

***L*-transformer ($f_L^\sharp$)**

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An abstract transformer $f^\sharp \in L$ is a **best L-transformer** (denoted as $\widehat{f_L^\sharp}$) if $f_L^\sharp$ is both sound and there does not exist any other transformer in L that is more precise.

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Abstract Interpretation Engines for Free!

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Now, let's dive into the workings of AMURTH.

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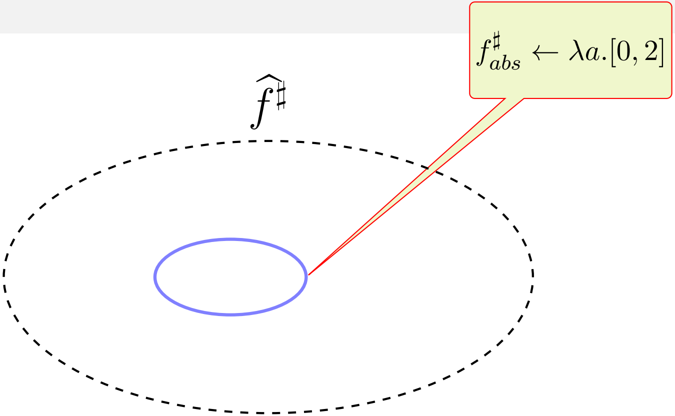
- Counterexamples generated by the **soundness** and **precision** verifiers drive two CEGIS loops.

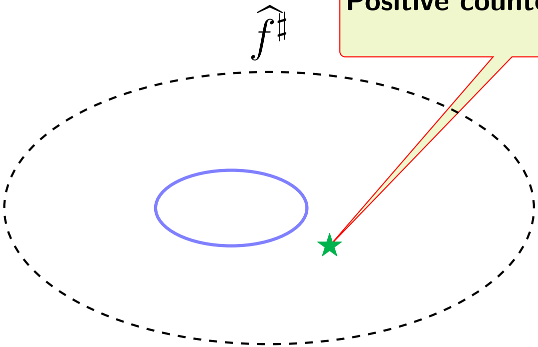
AMURTH in action!



AMURTH in action!

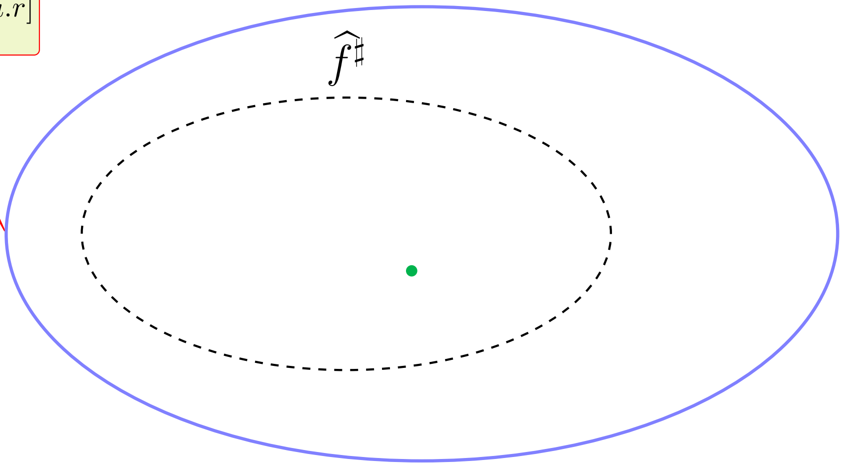
$$f^{\sharp}_{abs} \leftarrow \lambda a.[0, 2]$$

Positive counterexample: $\langle [0, 5], 3 \rangle$



Amurth in action!

$$f^\sharp_{abs} \leftarrow \lambda a.[0, a.l + a.r]$$



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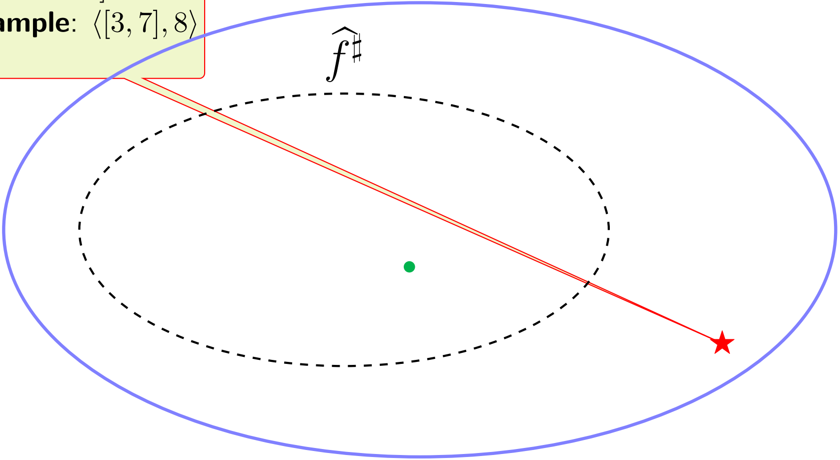
Negative counterexample: $\langle [3,7], 8 \rangle$



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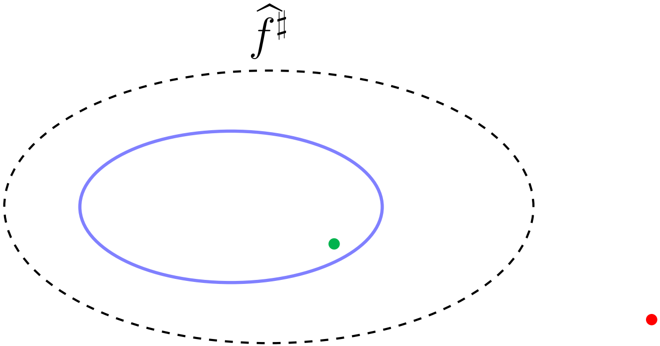

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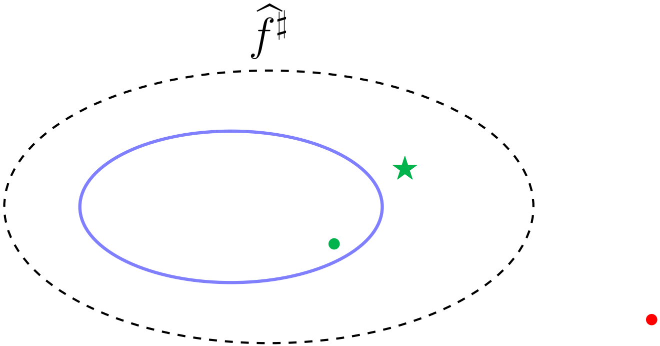

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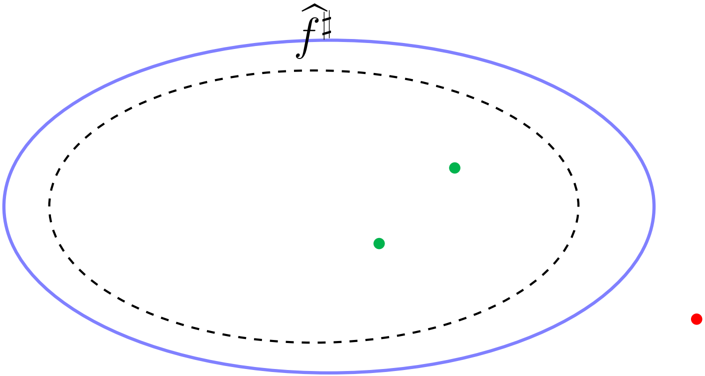

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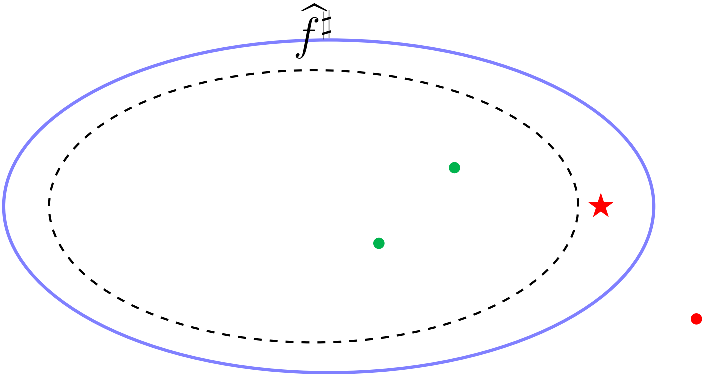

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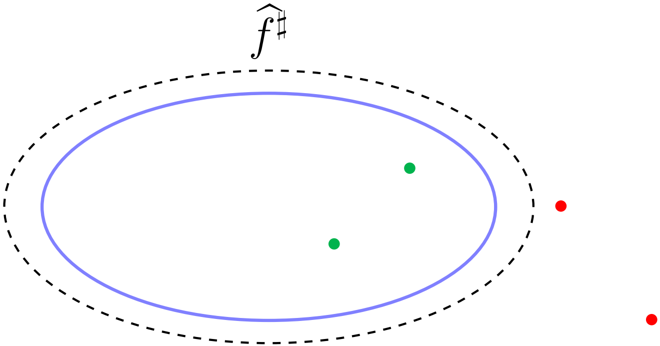

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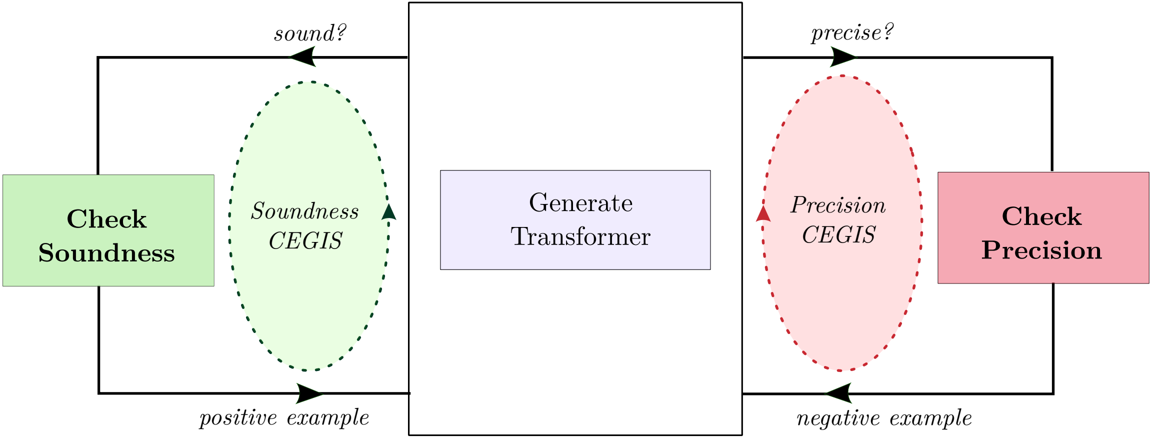
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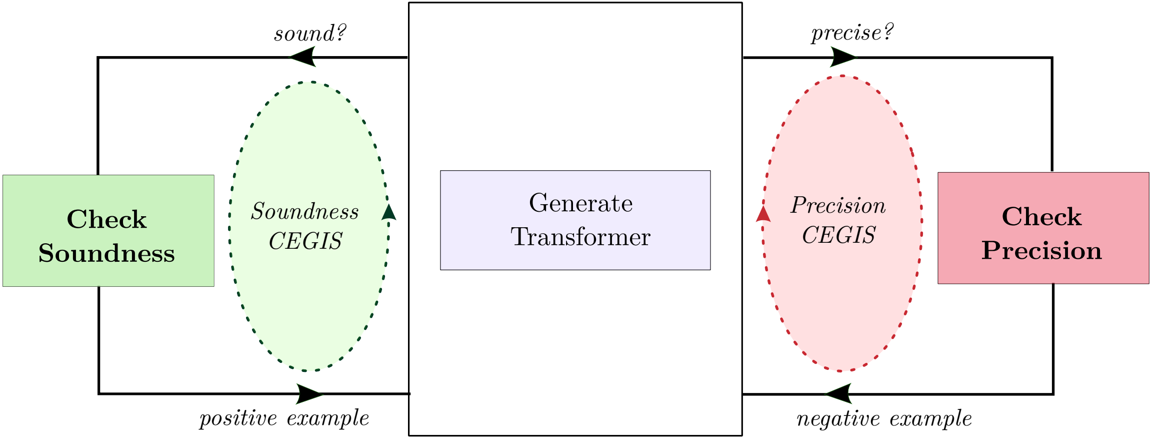

Additional algorithmic components are needed! (see the paper for details)

Claims

Theorem 1

If Algorithm terminates, it returns a best L-transformer for the concrete function f.

Theorem 2

If the DSL L is finite, Algorithm always terminates.

Evaluation: Domains and Operations

Domain Type	Abstract Domains	Operations
String	Constant String ($\mathcal{CS}$)	charAt$^\sharp$
	String Set (size k) ($\mathcal{SS}_k$)	concat$^\sharp$,
	Char Inclusion ($\mathcal{CI}$)	contains$^\sharp$,
	Prefix-Suffix ($\mathcal{PS}$)	toLower$^\sharp$, toUpper$^\sharp$,
	String Hash ($\mathcal{SH}$)	trim$^\sharp$
Fixed Bitwidth Interval	Unsigned-Int ($\mathcal{A}_{\texttt{uintv}}$)	add$^\sharp$, sub$^\sharp$, mul$^\sharp$,
	Signed-Int ($\mathcal{A}_{\texttt{uintv}}$)	and$^\sharp$, or$^\sharp$, xor$^\sharp$,
	Wrapped ($\mathcal{W}$)	shl$^\sharp$, ashr$^\sharp$, lshr$^\sharp$

Domains proposed in prior work.

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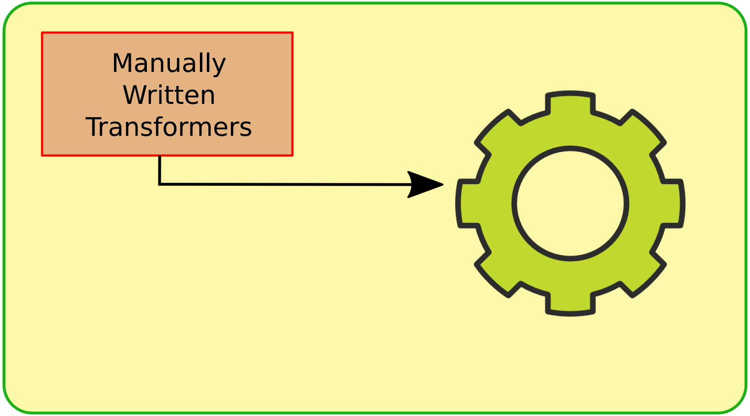
Evaluation: Methodology

(Existing) Abstract Interpretation Engine



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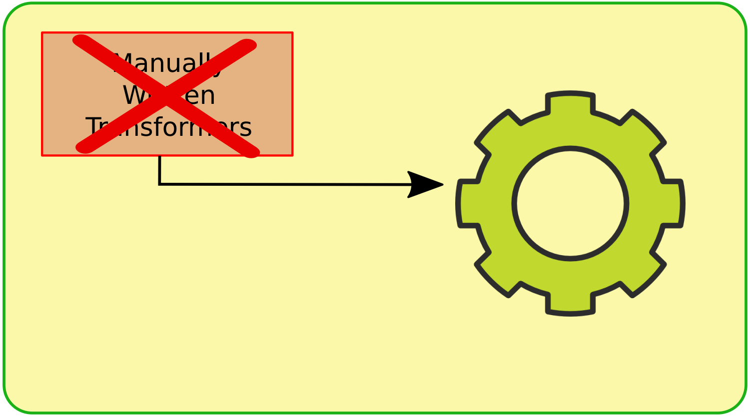

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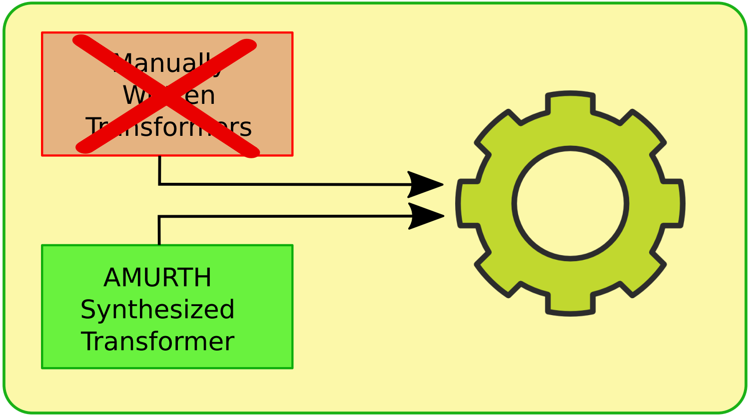
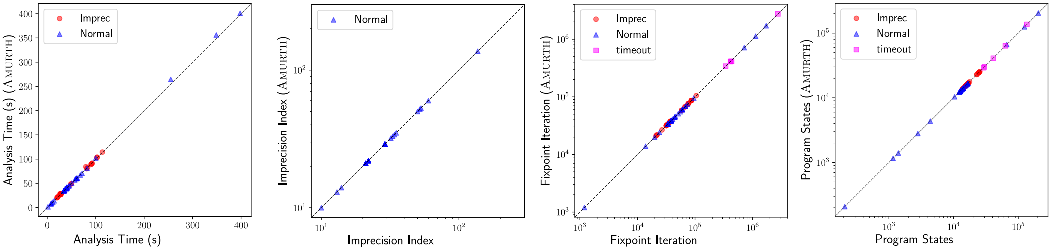


Comparison with manual operations



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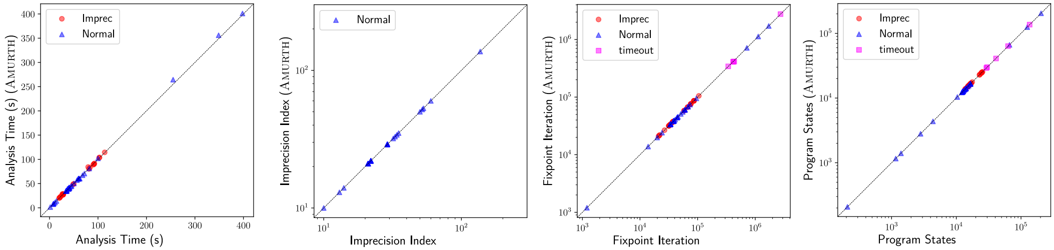

Similar performance as manually written transformers in terms of analysis time, imprecision index, fixpoint iteration, program states.

However, we discovered 4 soundness bugs in the manually written transformers.

Bug #1: contains in $\mathcal{CI}$

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$$\texttt{contains(str1, str2)} = \begin{cases} \top & \text{if } \texttt{str2} \text{ is contiguous substring of } \texttt{str1} \\ \bot & \text{otherwise} \end{cases}$$

Manually written transformer

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contains#_org(a1 : CI)(a2 : CI) : AbsBool =
  ite(isBot(a1.l, a1.u) ∨ isBot(a2.l, a2.u),
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    ite(isTop(a1.l, a1.u) ∨ isTop(a2.l, a2.u),
      boolTop,
      ite(¬isSubset(a2.l, a1.u),
        boolFalse,
          ite(size(a2.u) ≤ 1 ∧ isSubset(a2.u, a1.l),
            boolTrue,
            boolTop))))
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Synthesized abstract transformer

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contains#_syn(a1 : CI)(a2 : CI) : AbsBool =
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      ite(¬isSubset(a2.l, a1.u),
        boolFalse,
        ite(size(a2.u) ≤ 1 ∧ isSubset(a2.u, a1.l),
          boolTrue,
          boolTop))))
```

Synthesized abstract transformer

```
contains♯_syn(a1 : CI)(a2 : CI) : AbsBool =
  ite(isBot(a1.l, a1.u) ∨ isBot(a2.l, a2.u),
    boolBot,


      ite(¬isSubset(a2.l, a1.u),
        boolFalse,
        ite(isEmpty(a2),
          boolTrue,
          boolTop)))
```

s_1="aa", s_2="aaaaa"

a_1=[{'a'}, {'a'}], a_2=[{'a'}, {'a'}]

Conclusions

- Current techniques at handling such operations are either highly imprecise, unsound, or manual and error-prone.

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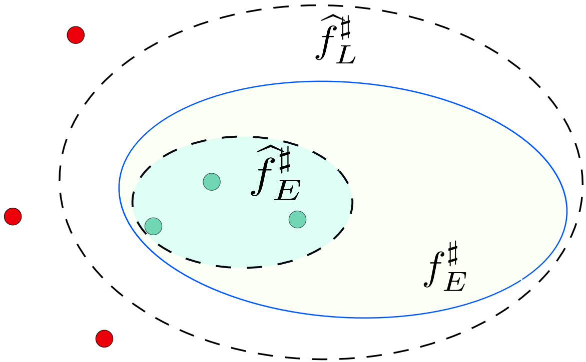
- Current techniques at handling such operations are either highly imprecise, unsound, or manual and error-prone.
- Our tool, AMURTH, is capable of automatically synthesizing non-trivial abstract transformers
- Our experiments on the existing tools shows the value of such an endeavour.

I thank Google for the generous travel grant that allowed me to attend SPLASH 2022.

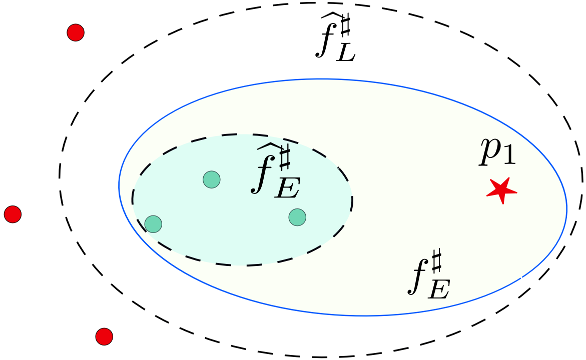
Thank you!

Backup slides

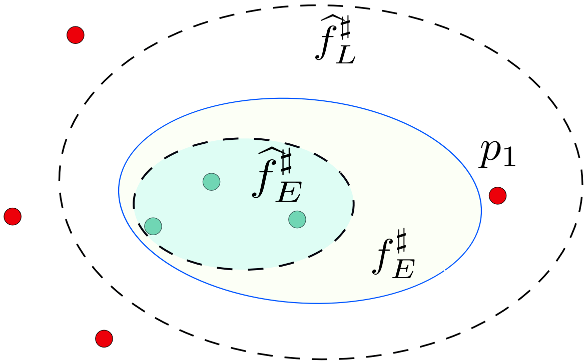
Failed consistency



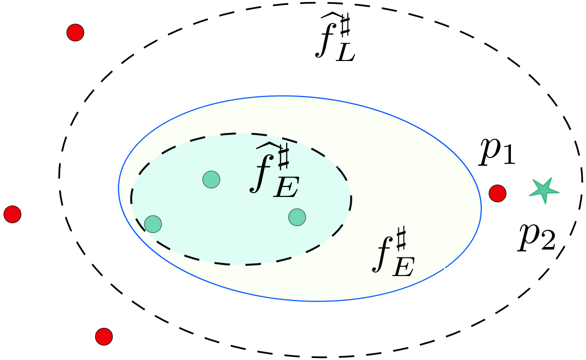
Failed consistency



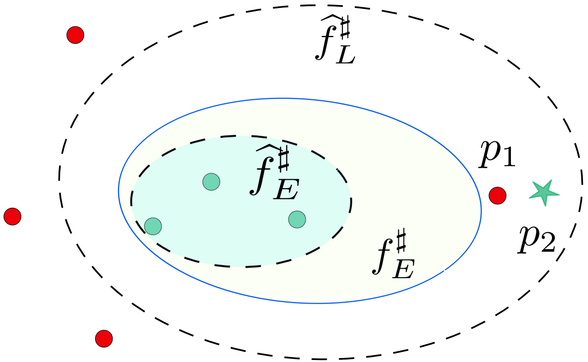
Failed consistency



Failed consistency

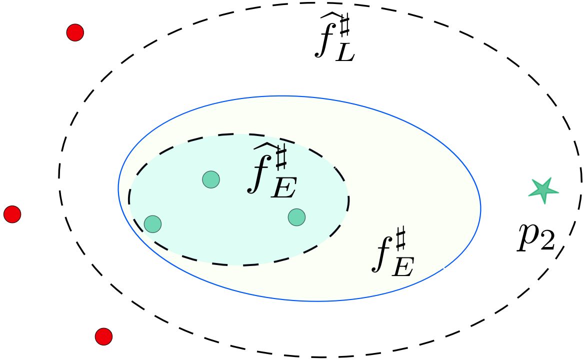


Failed consistency



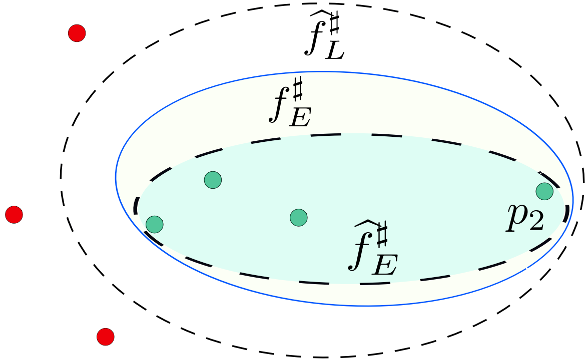
Inconsistent: no $f_E^\sharp \in L$ that satisfies all positive and negative examples.

Failed consistency



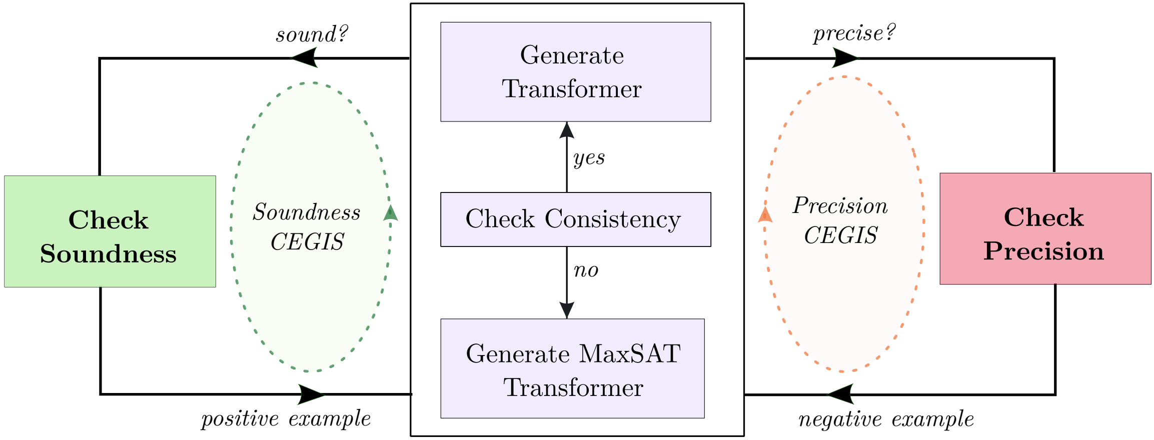
Occam's razor

Failed consistency



Occam's razor

Algorithm


sound?
precise?
Generate Transformer
yes
Check Soundness
Soundness CEGIS
Check Consistency
Precision CEGIS
Check Precision
no
Generate MaxSAT Transformer
positive example
negative example

trim in $\mathcal{CI}$

Original (buggy) transformer in SAFE_{str}

```
trim♯_org(a : CI) : CI =
 ite(isBot(a.l,a.u),
   boolBot,
   ite(isTop(a.l,a.u),
     boolTop,
     ite(size(a.u)≤1∧containsSpace(a.u),
       [∅,∅],
       a )))
```

Synthesized abstract transformer

```
trim♯_syn(a : CI) : CI =
 ite(isBot(a.l,a.u),
   boolBot,
   ite(isTop(a.l,a.u),
     boolTop,
     ite(size(a.u)≤1∧containsSpace(a.u),
       [∅,∅],
       [removeSpace(a.l),a.u] )))
```

strs = {"␣␣abc␣␣", "a␣a"}

$$a = [\{\text{`␣'},\text{`a'}\},\ \{\text{`␣'},\text{`a'},\text{`b'},\text{`c'}\}]$$

$$trim^\sharp(a) = [\{\text{`a'}\},\ \{\text{`␣'},\text{`a'},\text{`b'},\text{`c'}\}]$$