

Synthesis of Semantic Actions in Attribute Grammars

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Semantic Action

4 ■ 23
5 ♦ 9
34 ■ (12 ♦ 8)
(4 ♦ 7) ■ (5 ♦ 12)
...

Semantic Action

4 ■ 23
5 ◆ 9
34 ■ (12 ◆ 8)
(4 ◆ 7) ■ (5 ◆ 12)
...

$L \rightarrow E$
 $E \rightarrow E' \blacklozenge T$
 $E \rightarrow T$
 $T \rightarrow T' \blacksquare F$
 $T \rightarrow F$
 $F \rightarrow (E)$
 $F \rightarrow \text{digit}$

Semantic Action

4 ■ 23	92
5 ◆ 9	14
34 ■ (12 ◆ 8)	680
(4 ◆ 7) ■ (5 ◆ 12)	187
...	

L → E
E → E' ◆ T
E → T
T → T' ■ F
T → F
F → (E)
F → digit

Semantic Action

4 ■ 23	92
5 ◆ 9	14
34 ■ (12 ◆ 8)	680
(4 ◆ 7) ■ (5 ◆ 12)	187
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$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = E'.val + T.val$
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$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

Parsing of string

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F_1
|
4 \blacksquare 23

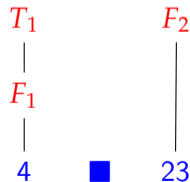
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T_1
|
 F_1
|
4 ■ 23

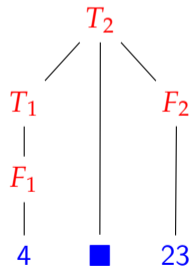
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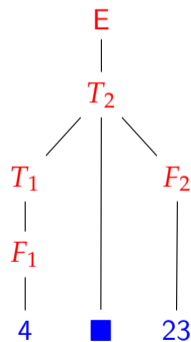
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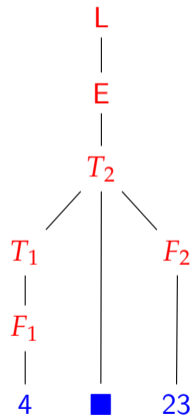
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Evaluation of strings

1. $L \rightarrow E$	$L.val = E.val$
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$F_1.val = 4$

4



23

Evaluation of strings

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$$T_1.val = F_1.val$$

$$F_1.val = 4$$

4

23

Evaluation of strings

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$$T_1.val = F_1.val$$

$$F_1.val = 4$$

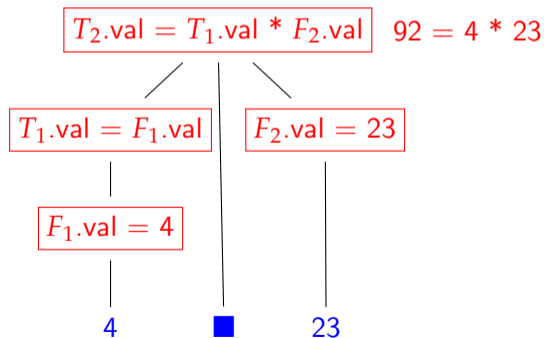
4

$$F_2.val = 23$$

23

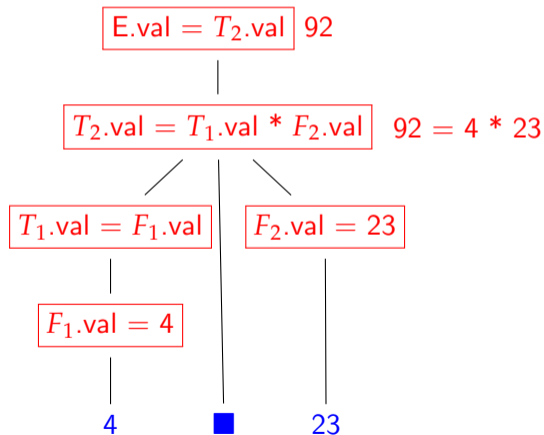
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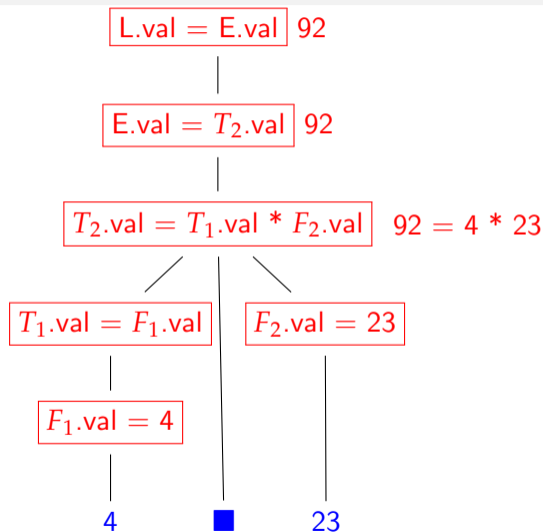
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Problem Statement

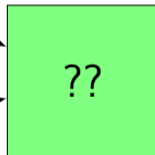
$L \rightarrow E$	
$E \rightarrow E' \blacklozenge T$	
$E \rightarrow T$	
$T \rightarrow T' \blacksquare F$??
$T \rightarrow F$	
$F \rightarrow (E)$	
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$4 \blacksquare 23$	92
$5 \blacklozenge 9$	14
$34 \blacksquare (12 \blacklozenge 8)$	680
$(4 \blacklozenge 7) \blacksquare (5 \blacklozenge 12)$	187

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$L \rightarrow E$??
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Grammar with Holes

$L \rightarrow E$	$L.val = E.val$
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Grammar with Holes

$L \rightarrow E$	$L.val = E.val$
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Grammar with Holes

Definition is missing.

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$T \rightarrow F$	$T.val = F.val$
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Problem Statement: Sketch Based Synthesis

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^*(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^*(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

DSL

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Problem Statement: Sketch Based Synthesis

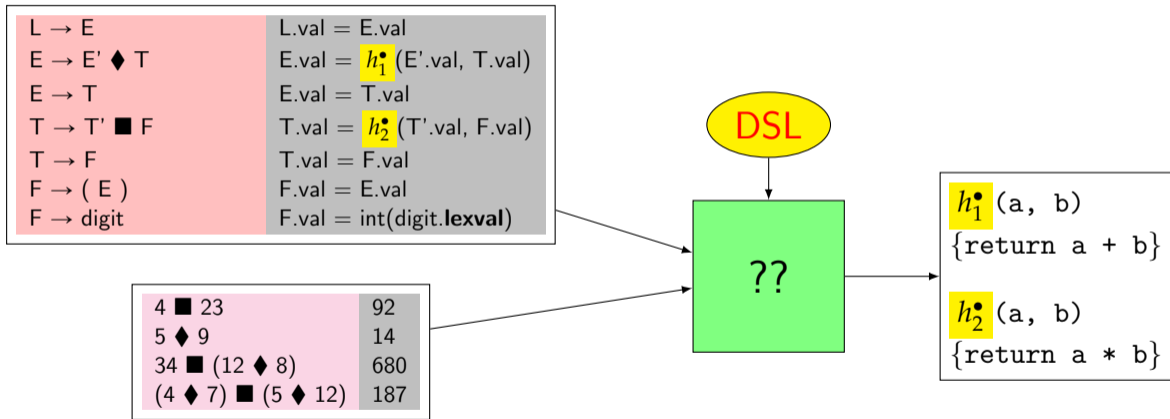
$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^*(E'.val, T.val)$
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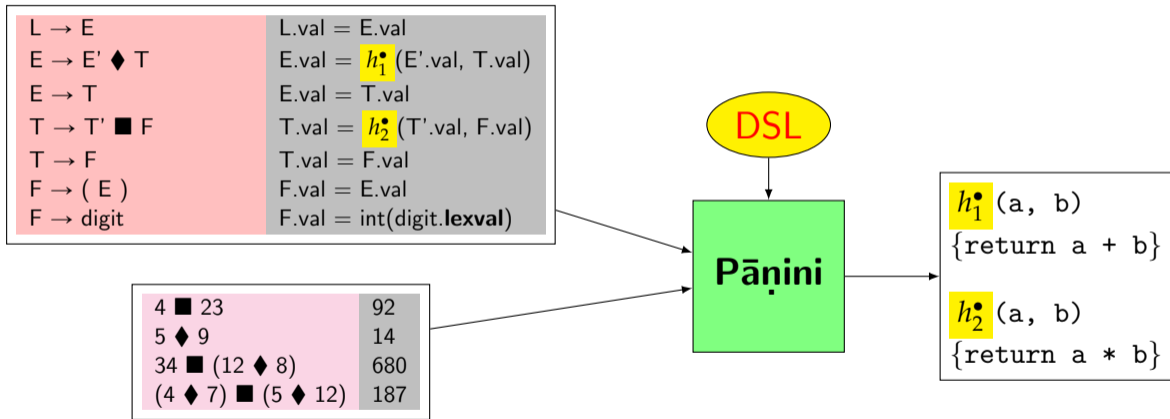
DSL

??

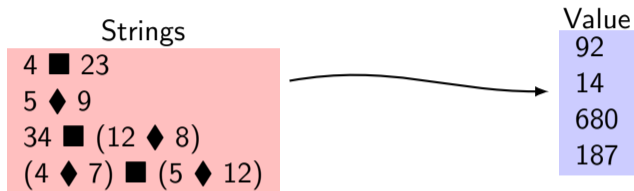
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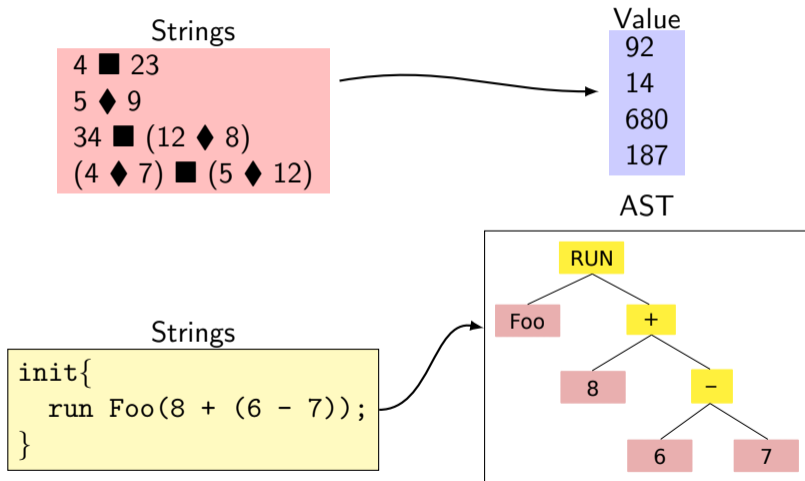
Problem Statement: Sketch Based Synthesis



Motivation



Motivation



Forward Differentiation

$$[[x+x]]_{x=13} = 26 + 2\varepsilon,$$

$$[[3-x]]_{x=7} = -4 - 1\varepsilon,$$

$$[[x*x]]_{x=4} = 16 + 8\varepsilon,$$

```
add (a1 + a2ε, b1 + b2ε):  
  r ← a1 + b1  
  d ← a2 + b2  
  return r + dε
```

(a)

```
sub (a1 + a2ε, b1 + b2ε):  
  r ← a1 - b1  
  d ← a2 - b2  
  return r + dε
```

(b)

```
mul (a1 + a2ε, b1 + b2ε):  
  r ← a1 * b1  
  d ← a2 * b1 + a1 * b2  
  return r + dε
```

(c)

```
sin (a1 + a2ε):  
  r ← sin(a1)  
  d ← a2 * cos(a1)  
  return r + dε
```

(d)

```
cos (a1 + a2ε):  
  r ← cos(a1)  
  d ← a2 * sin(a1) *  
    -1  
  return r + dε
```

(e)

```
pow (a1 + a2ε, c):  
  r ← pow(a1, c)  
  d ← a2 * pow(a1, c - 1)  
  return r + dε
```

(f)

Challenges: #1

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
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$T \rightarrow F$	$T.val = F.val$
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How to convert semantic action synthesis to program synthesis?



Solution: Symbolic Trace

4 ■ 23

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$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

4 ■ 23

Solution: Symbolic Trace

4 ■ 23 92

$F_1.val \leftarrow 4$

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
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F_1
|
4 ■ 23

Solution: Symbolic Trace

4 ■ 23 92

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$F \rightarrow (E)$	$F.val = E.val$
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$F_1.val \leftarrow 4$
 $T_1.val \leftarrow F_1.val$

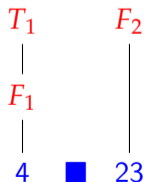
T_1
|
 F_1
|
4 ■ 23

Solution: Symbolic Trace

4 ■ 23 92

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$F_1.val \leftarrow 4$
 $T_1.val \leftarrow F_1.val$
 $F_2.val \leftarrow 23$

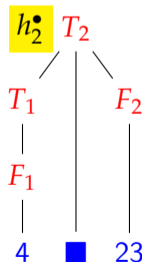


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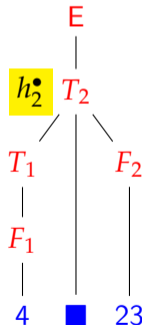
$F_1.val \leftarrow 4$
 $T_1.val \leftarrow F_1.val$
 $F_2.val \leftarrow 23$
 $T_2.val \leftarrow h_2^\bullet(T_1.val, F_2.val)$



Solution: Symbolic Trace

4 ■ 23 92

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$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

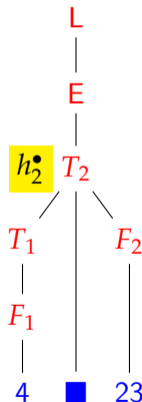


$F_1.val \leftarrow 4$
 $T_1.val \leftarrow F_1.val$
 $F_2.val \leftarrow 23$
 $T_2.val \leftarrow h_2^\bullet(T_1.val, F_2.val)$
 $E.val \leftarrow T_2.val$

Solution: Symbolic Trace

4 ■ 23 92

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

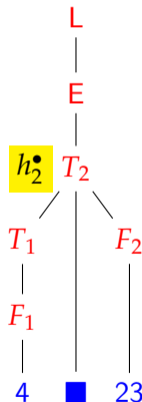


$F_1.val \leftarrow 4$
 $T_1.val \leftarrow F_1.val$
 $F_2.val \leftarrow 23$
 $T_2.val \leftarrow h_2^\bullet(T_1.val, F_2.val)$
 $E.val \leftarrow T_2.val$
 $L.val \leftarrow E.val$

Solution: Symbolic Trace

4 ■ 23 92

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

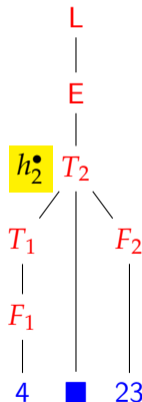


$F_1.val \leftarrow 4$
 $T_1.val \leftarrow F_1.val$
 $F_2.val \leftarrow 23$
 $T_2.val \leftarrow h_2^\bullet(T_1.val, F_2.val)$
 $E.val \leftarrow T_2.val$
 $L.val \leftarrow E.val$
output $\leftarrow L.val$

Solution: Symbolic Trace

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

4 ■ 23 92



$F_1.val \leftarrow 4$
 $T_1.val \leftarrow F_1.val$
 $F_2.val \leftarrow 23$
 $T_2.val \leftarrow h_2^\bullet(T_1.val, F_2.val)$
 $E.val \leftarrow T_2.val$
 $L.val \leftarrow E.val$
 $\text{output} \leftarrow L.val$
assert output = 92

Challenges: #2

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

How to use these programs to synthesize these holes?



Infinite Strings

```
4 ■ 23 = 92
```

```
F2.val ← 4
```

```
T2.val ← F2.val
```

```
F1.val ← 23
```

```
T1.val ←  $h_2^{\bullet}$ (T2.val, F1.val)
```

```
E.val ← T1.val
```

```
L.val ← E.val
```

```
output ← L.val
```

```
assert output = 92
```

Infinite Strings

4 ■ 23 = 92

F₂.val ← 4

T₂.val ← F₂.val

F₁.val ← 23

T₁.val ← h_2^\bullet (T₂.val, F₁.val)

E.val ← T₁.val

L.val ← E.val

output ← L.val

assert output = 92

5 ♦ 9 = 14

F₁.val ← 9

T₁.val ← F₁.val

E₁.val ← T₁.val

F₂.val ← 5

T₂.val ← F₂.val

E₂.val ← h_1^\bullet (E₁.val, T₂.val)

L.val ← E₂.val

output ← L.val

assert output = 14

Infinite Strings

$$4 \blacksquare 23 = 92$$

$F_2.\text{val} \leftarrow 4$

$T_2.\text{val} \leftarrow F_2.\text{val}$

$F_1.\text{val} \leftarrow 23$

$T_1.\text{val} \leftarrow h_2^\bullet(T_2.\text{val}, F_1.\text{val})$

$E.\text{val} \leftarrow T_1.\text{val}$

$L.\text{val} \leftarrow E.\text{val}$

$\text{output} \leftarrow L.\text{val}$

assert output = 92

$$5 \blacklozenge 9 = 14$$

$F_1.\text{val} \leftarrow 9$

$T_1.\text{val} \leftarrow F_1.\text{val}$

$E_1.\text{val} \leftarrow T_1.\text{val}$

$F_2.\text{val} \leftarrow 5$

$T_2.\text{val} \leftarrow F_2.\text{val}$

$E_2.\text{val} \leftarrow h_1^\bullet(E_1.\text{val}, T_2.\text{val})$

$L.\text{val} \leftarrow E_2.\text{val}$

$\text{output} \leftarrow L.\text{val}$

assert output = 14

...

$$34 \blacksquare (12 \blacklozenge 8) = 680$$

$F_{34}.\text{val} \leftarrow 34$

$F_{12}.\text{val} \leftarrow 12$

$F_8.\text{val} \leftarrow 8$

$E_1.\text{val} \leftarrow h_1^\bullet(F_{12}.\text{val}, F_8.\text{val})$

$T_1.\text{val} \leftarrow h_2^\bullet(F_{34}.\text{val}, E_1.\text{val})$

$E_2.\text{val} \leftarrow T_1.\text{val}$

$L_1.\text{val} \leftarrow E_2.\text{val}$

$\text{output} \leftarrow L_1.\text{val}$

Post:output = 680

Infinite Strings

$$4 \blacksquare 23 = 92$$

$F_2.\text{val} \leftarrow 4$

$T_2.\text{val} \leftarrow F_2.\text{val}$

$F_1.\text{val} \leftarrow 23$

$T_1.\text{val} \leftarrow h_2^{\bullet}(T_2.\text{val}, F_1.\text{val})$

$E.\text{val} \leftarrow T_1.\text{val}$

$L.\text{val} \leftarrow E.\text{val}$

$\text{output} \leftarrow L.\text{val}$

assert output = 92

$$5 \blacklozenge 9 = 14$$

$F_1.\text{val} \leftarrow 9$

$T_1.\text{val} \leftarrow F_1.\text{val}$

$E_1.\text{val} \leftarrow T_1.\text{val}$

$F_2.\text{val} \leftarrow 5$

$T_2.\text{val} \leftarrow F_2.\text{val}$

$E_2.\text{val} \leftarrow h_1^{\bullet}(E_1.\text{val}, T_2.\text{val})$

$L.\text{val} \leftarrow E_2.\text{val}$

$\text{output} \leftarrow L.\text{val}$

assert output = 14

...

$$34 \blacksquare (12 \blacklozenge 8) = 680$$

$F_{34}.\text{val} \leftarrow 34$

$F_{12}.\text{val} \leftarrow 12$

$F_8.\text{val} \leftarrow 8$

$E_1.\text{val} \leftarrow h_1^{\bullet}(F_{12}.\text{val}, F_8.\text{val})$

$T_1.\text{val} \leftarrow h_2^{\bullet}(F_{34}.\text{val}, E_1.\text{val})$

$E_2.\text{val} \leftarrow T_1.\text{val}$

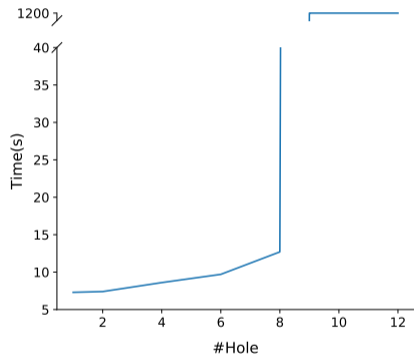
$L_1.\text{val} \leftarrow E_2.\text{val}$

$\text{output} \leftarrow L_1.\text{val}$

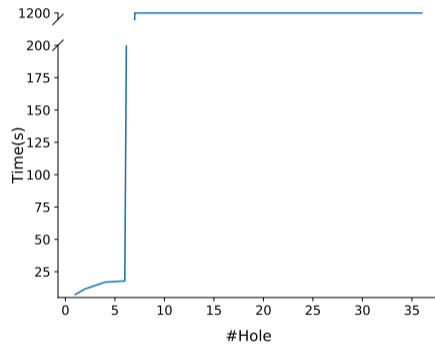
Post:output = 680

Two holes, i.e., h_1^{\bullet} and h_2^{\bullet} , are being shared among infinite strings.

Scalability

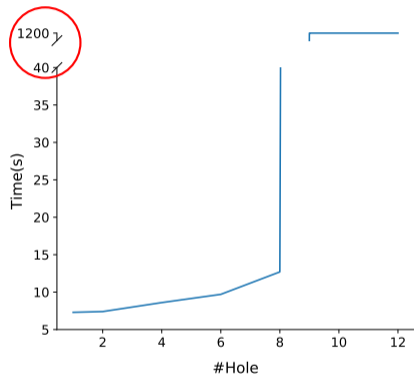


(a) Forward differentiation (b10)

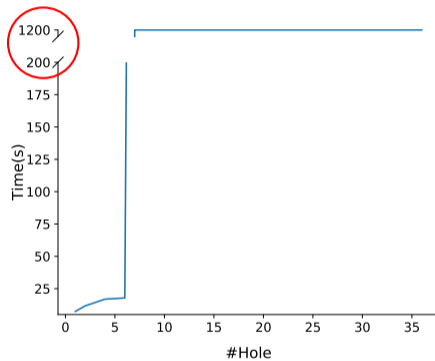


(b) Java bytecode (b11)

Scalability

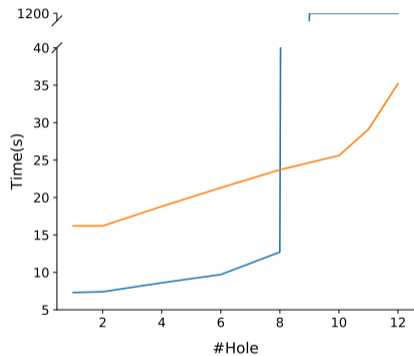


(a) Forward differentiation (b10)

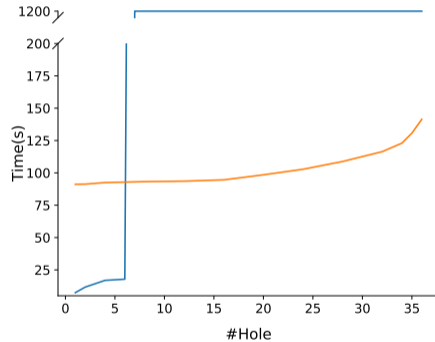


(b) Java bytecode (b11)

Incremental Synthesis to the Rescue



(a) Forward differentiation (b10)



(b) Java bytecode (b11)

PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = ??$$

$$h_2^\bullet(a, b) = ??$$

$4 \blacksquare 23$	92
$5 \blacklozenge 9$	14
$34 \blacksquare (12 \blacklozenge 8)$	680
$(4 \blacklozenge 7) \blacksquare (5 \blacklozenge 12)$	187

PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = ??$$

$$h_2^\bullet(a, b) = ??$$

4 ■ 23

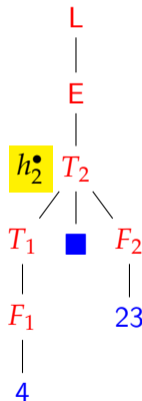
PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$h_1^\bullet(a, b) = ??$

$h_2^\bullet(a, b) = ??$

4 ■ 23



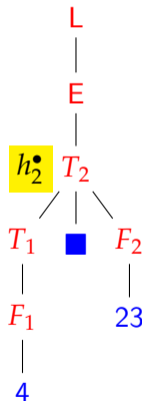
PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$h_1^\bullet(a, b) = ??$

$h_2^\bullet(a, b) = ??$

4 ■ 23 92



```
F1.val ← 4
T1.val ← F1.val
F2.val ← 23
T2.val ← h2^\bullet(T1.val, F2.val)
E.val ← T2.val
L.val ← E.val
output ← L.val
assert output = 92
```

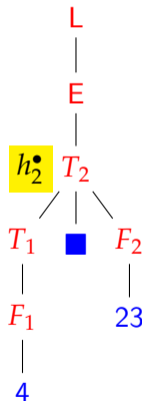
PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = ??$$

$$h_2^\bullet(a, b) = a * b$$

4 ■ 23 92



```
F1.val ← 4
T1.val ← F1.val
F2.val ← 23
T2.val ← h2^\bullet(T1.val, F2.val)
E.val ← T2.val
L.val ← E.val
output ← L.val
```

```
assert output = 92
```

$$h_2^\bullet(a, b) = a * b$$

PĀṆINI in Action

5 ♦ 9

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = ??$$

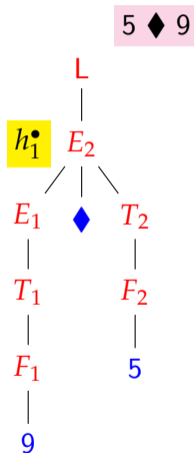
$$h_2^\bullet(a, b) = a * b$$

PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = ??$$

$$h_2^\bullet(a, b) = a * b$$

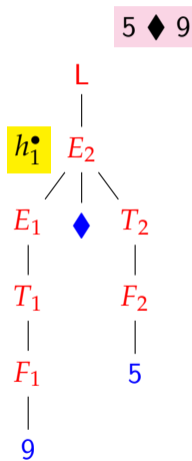


PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = ??$$

$$h_2^\bullet(a, b) = a * b$$



```

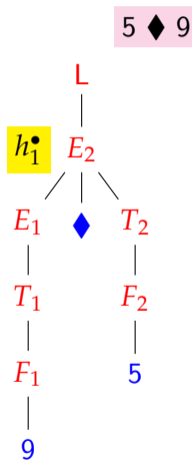
F1.val ← 9
T1.val ← F1.val
E1.val ← T1.val
F2.val ← 5
T2.val ← F2.val
E2.val ← h1^\bullet(E1.val, T2.val)
L.val ← E2.val
output ← L.val
assert output = 14
    
```

PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = a + a + a - 1$$

$$h_2^\bullet(a, b) = a * b$$



```

F1.val ← 9
T1.val ← F1.val
E1.val ← T1.val
F2.val ← 5
T2.val ← F2.val
E2.val ← h1^bullet(E1.val, T2.val)
L.val ← E2.val
output ← L.val
assert output = 14
  
```

$$h_1^\bullet(a, b) = a + a + a - 1$$

PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = a + a + a - 1$$

$$h_2^\bullet(a, b) = a * b$$

34 ■ (12 ♦ 8) 680

```
F34.val ← 34
F12.val ← 12
F8.val ← 8
E1.val ← h1•(F12.val, F8.val)
T1.val ← h2•(F34.val, E1.val)
E2.val ← T1.val
L1.val ← E2.val
output ← L1.val
assert output = 680
```

PĀṆINI in Action

34 ■ (12 ♦ 8) 680

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = a + a + a - 1$$

$$h_2^\bullet(a, b) = a * b$$

$F_{34}.val \leftarrow 34$

UNSAT (al)

$T_1.val \leftarrow h_2^\bullet(F_{34}.val, E_1.val)$

$E_2.val \leftarrow T_1.val$

$L_1.val \leftarrow E_2.val$

$\text{output} \leftarrow L_1.val$

assert output = 680

PĀṆINI in Action

34 ■ (12 ♦ 8) 680

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = a + a + a - 1$$

$$h_2^\bullet(a, b) = a * b$$

$F_{34}.val \leftarrow 34$

$F_{12}.val \leftarrow 12$

$F_8.val \leftarrow 8$

$E_1.val \leftarrow h_1^\bullet(F_{12}.val, F_8.val) \quad h_1^\bullet(12, 8) = 35$

$T_1.val \leftarrow h_2^\bullet(F_{34}.val, E_1.val) \quad h_2^\bullet(34, 35) = 1190$

$E_2.val \leftarrow T_1.val$

$L_1.val \leftarrow E_2.val$

$\text{output} \leftarrow L_1.val$

assert output = 680

PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$h_1^\bullet(a, b) = ??$

$h_2^\bullet(a, b) = ??$

34 ■ (12 ◆ 8) 680

5 ◆ 9 14

4 ■ 23 92

$F_{34}.val \leftarrow 34$

$F_{12}.val \leftarrow 12$

$F_8.val \leftarrow 8$

$E_1.val \leftarrow h_1^\bullet(F_{12}.val, F_8.val)$

$T_1.val \leftarrow h_2^\bullet(F_{34}.val, E_1.val)$

$E_2.val \leftarrow T_1.val$

$L_1.val \leftarrow E_2.val$

$\text{output} \leftarrow L_1.val$

assert output = 680

PĀṆINI in Action

$L \rightarrow E$	$L.val = E.val$
$E \rightarrow E' \blacklozenge T$	$E.val = h_1^\bullet(E'.val, T.val)$
$E \rightarrow T$	$E.val = T.val$
$T \rightarrow T' \blacksquare F$	$T.val = h_2^\bullet(T'.val, F.val)$
$T \rightarrow F$	$T.val = F.val$
$F \rightarrow (E)$	$F.val = E.val$
$F \rightarrow \text{digit}$	$F.val = \text{int}(\text{digit.lexval})$

$$h_1^\bullet(a, b) = a + b$$

$$h_2^\bullet(a, b) = a * b$$

34 ■ (12 ◆ 8) 680

5 ◆ 9 14

4 ■ 23 92

$F_{34}.val \leftarrow 34$

$F_{12}.val \leftarrow 12$

$F_8.val \leftarrow 8$

$E_1.val \leftarrow h_1^\bullet(F_{12}.val, F_8.val)$

$T_1.val \leftarrow h_2^\bullet(F_{34}.val, E_1.val)$

$E_2.val \leftarrow T_1.val$

$L_1.val \leftarrow E_2.val$

$\text{output} \leftarrow L_1.val$

assert output = 680

Case Study: Constant folding in SPIN

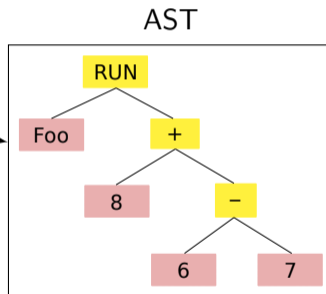
```
init{  
  int flags[(5 * 25) - 42];  
  int v = flags[10 - 4 + (9 / 3)];  
}
```

```
init {  
  int flags[83];  
  int v = flags[9];  
}
```

Case Study: PROMELA AST

Strings

```
init{  
  run Foo(8 + (6 - 7));  
}
```



THANK YOU



Extended version of this work.